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Ideality and Cognitive Development: Further Comments on Azeri's "The Match of Ideals"

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In his recent article, Siyaves Azeri (2020) responds to “Wigner’s riddle,” i.e., how it is that mathematics can be so “unreasonably” effective at explaining physical law, especially considering how so many advanced mathematical concepts—unlike those of elementary arithmetic and geometry—originate so far removed from the world of practical concern. Wigner’s answer, we learn, is in effect a non-answer: “the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve,” and amounts to nothing short of a “miracle” (Wigner 1960, 12).

Azeri rightly rejects such an outlook, which he claims is founded on “the contemplative stance”—the mistaken though historically entrenched idea that human cognition is isolated in the individual, consists in processes internal to the individual, and must somehow adequate to the external world—and demonstrates how so many responses to “Wigner’s riddle” (e.g., Hamming 1980, Wilson 2000, Colyvan 2001, Islami 2017) do no better than recapitulate internalistic and individualistic biases associated with a naïve materialism.

Even the recent work of Islami and Wiltsche fares no better for Azeri as it purports physics to be “a mathematized science in the sense that, at its core, it deals with idealized, exact objects—objects that are nowhere to be found in our ordinary experience of the world” (Azeri 2020, 8). Such a conception of ideality, Azeri claims, only reifies cognition as occurring in some “distinct and discrete realm ... in contradistinction to the practical and empirical realm” (6). Azeri explains that rather than being understood as some mental entity opposing the physical world, the ideal should instead be thought of as an orienting property of the social-objective world as interpreted and acted upon through one’s thoughts and actions:

The concept, or the ideal, constitutes norms of human activity in the form of schemata of action, which are to be internalized so that individual consciousness takes shape. The gradual appearance of concepts is observable in the process of ontogenesis of consciousness. A brief look at the history of sciences reveals that concepts are also subject to a similar *phylogenetic* [emphasis added] developmental process, i.e. they form and emerge, change, even sometimes die out against a socio-historical background. Mathematical concepts, contrary to the widespread belief in their trans-historicity, are no exception; they develop historically and are closely interconnected with the mode and the consequent schemata of human activity (12).

First and foremost, I think Azeri presents a stimulating and much needed Marxist critique of the supposed ideality of mathematics and its “miraculous” correspondence to physics. In his discussion, Azeri focuses mainly on the work of Ilyenkov, who saw his project as a continuation of Soviet Marxism inaugurated in the cultural historical psychology of L.S. Vygotsky and formalized in the activity theory of A.N. Leontiev. I see nothing wrong with this. Azeri’s paper, I believe, is a successful one. I would like to take the opportunity, however, to expand the scope of Azeri’s discussion by attending to the *phylogenetic* dimension of the development of conceptual cognition in general.

Azeri quite well shows that arithmetical thinking emerges on the basis of specific social practices and material engagement (clay tokens for economic exchange practices beget number concepts, e.g.). He also notes the normative basis of such thinking when he writes:

The invention-exploration of new areas of activity, that is, the expansion of the range of social (humanized) reality and the ‘thinkable’, bring about new norms and schemata that do not simply reside on the top of the formerly produced and accumulated schemata but rather subject them to qualitative changes, turning them into moments of these newly constituted norms (13).

But his discussion here is relegated mostly to Neolithic and Bronze Age practices. While surely such practices produced revolutions in the cognitive abilities of many humans, much of the cognitive architecture that allows normative conceptual thought was already in place long before this time. This response, then, is an attempt to sketch the deep prehistory of human cognition in order to show the inter-social bases of normative thought in general.

To do this, I will look first to the work of Vygotsky and Leontiev, two often neglected psychologists whose combined efforts culminate in a developmental account of human cognitive origins. Then, I will review some key insights from the contemporary comparative psychologist Michael Tomasello—whose project is admittedly a Vygotskian one—in order to further shed light on the social-practical basis of abstract thought, of which mathematical cognition is surely a part.

Vygotsky: Culture, Mediation, Internalization

Soviet developmental psychologist L.S. Vygotsky (1896-1934) is known for offering a dialectical and non-reductionistic account of human development as grounded in “material collaborative social practices” (Stetsenko and Arieviditch 2010, 231). Writing in the years following the October Revolution, Vygotsky was critical, *inter alia*, of the outdated metaphysical presuppositions of early 20th century psychology. His most basic methodological concern was to formulate a “general psychology” that could move beyond the stale impasse between “Objective” and “Subjective” programs in psychology. In many ways, such an impasse was a holdover from early modern debates between monistic materialism and substance dualism.

Fast-forward to the early 20th century and you had either “Subjectivist” theories of psycho-physical parallelism attempting to bridge the mind-body divide, or like the “Objectivist” you could reject the mental all together in the name of reflex. For Vygotsky, neither route would do. Such a divide must be bridged along a third “way”—by studying the relation of the individual to her social environment.

For Vygotsky, the socio-material world is an integral component in the cognitive development of a given individual. To study cognition is thus to study its *development*. And this necessitates taking into consideration an individual’s *cultural-historical* context. Vygotsky accomplishes this by recourse to a multi-scalar “genetic” methodology which takes as its

object the ontogenesis, phylogenesis, and historical emergence of particularly human cognitive capacities.

However, a disambiguation must be made regarding the relationship between phylogenetic and historical development. While a “Vygotskian” program avails itself of any “deep” evolutionary data to explain mental functioning, with respect to the question of human cognitive origins *phylogeny* must be considered to be culturally and artifactually determined: human history begins with technical-semiotic development and the concurrent development of cultural forms of life. In other words, we could say that for Vygotsky not only is the general biological development of the species taken as explanatorily relevant with respect to cognition, but more importantly is the species’ development *as it leaves* the bounds of purely biological determination.

As Vygotsky explains, the “historical development of man differs from the adaptation and development of animals because the process of mental development in man is part of the total historical development of mankind” (Vygotsky 1997b, 39). Such a general level of explanation in part informs further developmental levels: the ontogenetic level (the individual history of a subject), and the specific history of the development of higher mental functions (which draws from general cultural histories to explain the development of particular psychological systems). For our purposes, the phylogenetic level must somewhat be collapsed with the cultural-historical. This is important for our reading of Tomasello, which we will see provides a cultural/phylogenetic explanation of the emergence of the kind of abstract cognition necessary for the creation of mathematical concepts.

According to Vygotsky, psychological development is the result of “internalization,” or the functional reorganization of cognitive capacities based on socio-material mediation. This is the defining feature of Vygotsky’s application of Marxian materialism to psychology: that the human mind is a historical rather than biological product, one based ultimately on the labor activity of humans in social groups. Distinctly human “higher mental functions” (e.g., semantic memory, propositional cognition, numeracy, and literacy) arise during the organism’s transition away from the biologically primary behavior of environmental and ecological affordance-based reactions to the culturally and technically mediated activity of the self-conscious agent. Such an agent is a self-regulating entity which has mastered its “reactive functions” and as such can enter into new and varied goal-directed relations with its environment based on meaningful (semiotic), rather than simply functional, constraints:

In subjecting to his will the process of his own reactions, man enters in this way into a substantially new relation with the environment, comes to a new functional exploitation of elements in the environment as stimulus-signs which he uses... and directs and controls his own behavior... and elicits reactions that he desires (Vygotsky 1999, 63).

The mediation of activity through technical and semiotic devices or signs—i.e., psychological tools or “cognitive artifacts” (Norman 1991)—is thus the main effector of the structural and functional transformation of cognitive capacities. It is precisely these devices as socio-

cultural “repositories” which effect, through socialized interaction with them, the genesis and structuration of higher cognitive abilities. Thus, when considered on an evolutionary timeline, the utilization of technical and psychological devices constitutes the cultural mechanism of hominization, accounting for the phylogenetic development of *Homo sapiens* as a historical species. When considered on an ontogenetic time-line, such psychological tools account for the emergence of higher mental functions insofar as they constitute the mechanisms accounting for the emergence of an individual agent via the internalization of once exterior social activity.

Mathematical cognition by Vygotsky’s score is a “higher mental function.” What we have seen thus far is that the emergence of such functions is predicated on cultural development. In this next section we’ll look more closely at the work of Vygotsky’s disciple, A.N. Leontiev, who emphasizes the normative dimension of cognition and its origin in joint activity. As Azeri correctly demonstrates, the ideal objects of mathematics are necessarily derived from human activity. With Leontiev, will we touch further on the phylogenetic emergence of such “ideal cognition.”

Leontiev: Activity, Ideality, and Phylogenesis

A.N. Leontiev (1903-1979) is known as the formalizer of ‘activity theory,’ a research program in psychology that seeks to explain the origins of human consciousness in terms of the practical activity of the socialized individual.¹ Following Marx and Vygotsky, Leontiev understands “object-oriented activity” as a central, unifying, category that explains the production of the social world and its “reflection” in human consciousness. That is, through activity, the human both *objectifies* the natural world in production, and in doing so, psychologically *appropriates* the various and interconnected social and objective relations laid forth in objectification. While Vygotsky stresses the importance of mediation in the genesis of higher mental functions, by Leontiev’s reading, his account comes up short in recognizing the contributions of the broader social world in the process of psychogenesis. Thus, Leontiev expands Vygotsky’s account to include the normative and collective goals of social systems at large.

According to Leontiev, insofar as the world of the developing individual is a product of cultural, historical, and material activity, that world is thus an *ideal* world. But Leontiev understands ideality in contradistinction to the traditional Western canon, where “ideal” connotes “mental” as opposed to “physical.” Rather, for Leontiev the ideal is part and parcel to the subject’s activity in the world. Taking Vygotsky’s insight that the origin of higher

¹ I would say ‘founder’ of the program if Vygotsky’s role as its antecedent was not so large. Most activity theorists, Leontiev included, take Vygotsky’s genetic methodology and his principle of the transformative power of cultural mediation as foundational to the discipline (Maidansky and Oittinen 2017). Regarding the concept of activity in general—though seen by many as its most influential theorist, the conception of activity as *Lebenstätigkeit* was not unique to Leontiev. It was common to many Soviet theorists who wished to contribute to a Marxian account of human consciousness and social life. Lektorsky, Ilyenkov, and Yudin are just some of the figures that took up the notion, and the Soviet intellectual scene of the 1960s and 70s was rife with competing philosophical-psychological theories of activity.

mental functions is to be found in the “equipped (‘instrumented’) structure of human activity and its incorporation into the system of interrelationships with other people,” Leontiev argues that such mental functions “assume a structure that has as its obligatory link socially-historically formed means and methods” (Leontiev 2009a, 94-95).

Leontiev further argues that in such processes of interiorization, “simultaneously there takes place a change in the very form of the psychological reflection of reality: Consciousness appears as a reflection by the subject of reality, his own activity, and himself” (95). Human consciousness is thus unique insofar as its content is ideal, relating to the broader social world. Yet such content remains vitally linked to a material basis insofar as its continued ontological maintenance depends on the social-practical engagement of the subject in her world. When taken in its specifically (human) psychological sense, activity thus refers not simply to “brain-processes,” but to social activity as refracted through the individual, both internally and externally.

For Leontiev, mental reflection and representations are dialectical phenomena, maintaining an existence “inseparable from the subject’s activity” (Leontiev 2009a, 76). Such phenomena are by no means “copies” of the objective world but rather exist as neuronal configurations that source potentials of orientation in the world; that is, the subjective image is objectively realized “as it becomes apparent to the person in one system or another of objective connections” (loc. cit.). However, Leontiev argues that the particularly human form of such reflective processes deals with the objective world as a social world, one that is itself the product of historical and cultural development.

The content of the reflected image, then, for the human is something meaningful in a manner not available to other forms of animal life: “Sensory images represent a universal form of psychic reflection having its origin in the objective activity of the subject” (Leontiev 2009a, 125). Thus, human cognition is differentiated from the non-human with respect to its social-semiotic content: “In man, however, sensory images assume a new quality, specifically, their signification. Meanings are the most important ‘formers’ of human consciousness” (loc. cit.). For Leontiev, then, whatever “meaningful” activity the non-human animal partakes in is still grounded on biological impulse. Only the human can entertain socially reflected i.e., *idealized*, meanings.²

Still, it’s important to remember that for Leontiev such idealizations must not be thought of as “emanations” of *Geist*. Not even logical and mathematical concepts (and by extension, the concepts of modern physics), despite the fact that they are still often “presented as some sort

² Leontiev (like Vygotsky before him) is rather traditional on this front, maintaining a clear nature/culture dichotomy in which non-natural meaning is available only to cultural animals (humans). But see Jablonka and Lamb (2005, 160) and Tomasello (2014, 82) for the view that culture (and thinking) abounds in the animal kingdom. Still, at least for Tomasello, non-human animal culture including that of animals evolutionarily closest to us (i.e., non-human primates) is essentially individualistic and exploitative. In contrast, the transmission and creation of human-specific culture is founded on the cooperative and recursive sociality endemic to the foraging niches of late Pliocene and early Pleistocene hominins, a sociality that serves as the basis for all further human cultural development.

of divine inspiration with no relation to social reality” (Azeri 2020, 5). Leontiev (2009a) argues instead that:

In contrast to the views of the laws of logic as if they arise from the principles of the working of the mind (or as if they express immanent laws of a thinking spirit, or finally as if they are evoked by the development of the language of science itself), the Marxist view considers logical laws as representing a generalized reflection of those objective relations of activity that practical human activity produces and to which it is subject. “The practical activity of man,” notes V. I. Lenin, “must have brought the consciousness of man a million times to the repetition of various logical figures in order that these figures might acquire the significance of axioms.” Thus, practical activity, practice, is like a guiding thread for theoretical thought that prevents theoretical thought from losing the way leading to adequate knowledge (55-56).

So practical activity breeds abstract cognition. That’s all well and good, but we are still after an explanation of how the capacity to entertain ideal content *phylogenetically* emerges. Leontiev fortunately does give us some details here. And while a full reckoning of Leontiev’s account is beyond the scope of this response (though see Drain 2018), we can note that Leontiev, like Vygotsky before him, accepts and refines Engels’ account of the role of labor in the phylogenetic development of human cognition (Engels 1946). Yet Leontiev goes beyond Engels in claiming that early humans only became humans at all by means of *joint activity*. Thus, non-human animals may engage in activity that is no more than an *aggregate* of individual activities, while humans enjoy a qualitatively different type of *joint* activity—they can do things *together*.

Leontiev’s account is close in spirit to Michael Tomasello’s (2014) recent work which traces the development of human cognition and communication along shifts in the social role of intentionality in early humans. Unfortunately, Leontiev does not explain exactly what is entailed by joint activity beyond mentioning that it is a uniquely human phenomenon: “we can observe the activity of a few, sometimes of many, animals together, but we never observe joint activity among them, i.e. in the sense of the word as we employ it when speaking of men’s activity” (2009b, 176). This is not particularly helpful—one would at least hope to hear what cognitive capacities and mechanisms are entailed by such joint activity but Leontiev is mute here. At least partially to blame is Leontiev’s avoidance of the concept of intentionality. Leontiev follows Vygotsky on this point and, though the reasons may be speculative, a case can be made that the vocabulary of “intentionality” is simply the currency of phenomenologists, especially Husserl, and should on those grounds alone be avoided. For Vygotsky, Husserl was one among a group of “extreme idealists, who reject the possibility of psychology as a natural science” (1997a, 294). As we saw with Azeri’s reading of Islami and Wiltsch, there may be good reason to avoid a phenomenology that reifies the ideal as something removed from socially embedded activity. But that doesn’t mean we must avoid intentionality altogether. As Tomasello shows, it is a cognitive feature that plays an essential role in the emergence of abstract thought.

Tomasello: Individual Intentionality, Protological Inferences, and Joint Intentionality

Michael Tomasello is a leading promulgator of the idea that social and cultural cooperation are primarily responsible for the evolutionary development of higher mental functions in humans. Tomasello analyzes the emergence of human cognition in a series of three phases, corresponding to creatures who are (1) individually intentional (the great apes), (2) jointly intentional varieties of pre-*sapiens* hominins, (*Homo erectus* and *Homo heidelbergensis*, e.g.), and (3) collectively intentional *Homo sapiens*. Central to Tomasello's account, then, is the idea that the main drivers of human cognitive differentiation rest uniquely on collaborative activity and shared intentionality.

As Tomasello argues, great apes and presumably a variety of pre-*sapiens* hominins had a range of cognitive skills that centered on the needs of the *individual*. Such beings indeed are “thinking beings” and entertain a variety of intentional states. Yet such primate intentionality is individually driven. Great apes and early hominins thus entertain representations—though not the socially contentful representations of the sort exemplified by Leontiev's “ideal reflection.” Rather, they represent relevant situations imagistically and schematically. A situation is relevant insofar as it affords either an opportunity or obstacle for personal gain (Tomasello 2014, 11). But how the organism decides to act in such a situation is not simply instinctually determined—at least for “higher” animals. Many organisms, primates included, go cognitively “beyond” the situation at hand in order to make creative inferences about what to do next.

Azeri mentions a remark by R.W. Hamming concerning the origin of mathematical reasoning in prehistorical humans (Azeri 2020, 4). To quote Hamming (1980) in full:

It seems ... that in the very foundations of primitive life there was built in, for survival purposes if for nothing else, an understanding of cause and effect. Once this trait is built up beyond a single observation to a sequence of ‘If this, then that, and then it follows further still that...,’ we are on the path of the first features of mathematics ... (83).

This comment proves a useful cipher in explaining the phylogenetic origin of abstract cognition in a Vygotskian vein. For as Tomasello shows, protological reasoning is a skill *already available to pre-human anthropoids*. That is, many great apes are adept at causal reasoning and can cognize about the social and physical world using “protological” inferences—a primitive sort of *modus ponens* and *tollens* (Tomasello 2014, 17-24). They can also engage in types of behavioral and cognitive self-monitoring in which they “seem to be monitoring not just imagined actions and their imagined results, or imagined causes and their imagined outcomes, but also their own knowledge or memory—which they then use to make inferences about their likelihood of behavioral success” (25). The point for us, however, is that such hypothetical reasoning can't be the proximate basis for the development of mathematical thought, because it's plenty common in animals who don't have mathematical

concepts. Something else needs to be added. And not *just* the element of social cognition, but social cognition as manifest in *cooperative* rather than individualistic creatures.

By Tomasello's account, organisms with individual intentionality are open to both causal and intentional inferences based on their previous perceptual experiences, and a situation may be relevant in terms of social and/or physical perceptual cues. For example, an ape may infer that a stick (that she wants to use) is being weighed down by a rock based on previous, now schematized, dealings with rocks. And after inferring or simulating a plan of action to remove the rock, she may dislodge the stick. Likewise, the ape may infer that a conspecific is "climbing a tree as an intentional event of a particular type, and then infer something about goals and attention as intentional causes as so predict the climber's impending actions" (Tomasello 2014, 13). The point is, such organisms schematize based on previous perceptual experiences in order to represent and simulate possible patterns of action. And as Tomasello notes, such simulations have a logical structure based on causal inferences. When dealing purely with the physical world, causal "if, then" reasoning drives the intentional action of the agent. In a relevant *social* situation, such causal reasoning is compounded with intentional reasoning. As Tomasello makes clear, *great apes think about the social world*. However, such cognition is based on competition (for food, sexual partners, and other resources) rather than cooperation.

Invoking Bakhurst's (2017) reading of Ilyenkov, Azeri (2020) remarks that:

The ideal is normative in character; it determines human thinking and actions rationally more than causally. Human activity is essentially guided by reason rather than being merely determined by causes. The ideal is independent of the will and thinking of the thinking subject; in this sense, it has an objective existence ... (11).

I think it's not much of a stretch to situate Tomasello's account of individual intentionality as occurring at a non-ideal, subjective/causal level. Whence comes the ideal, then, and its concomitant objectivity and rationality? For Tomasello the answer lies in the transition from individual to joint intentionality, where for the first time there emerges a cognitive-communicative shift towards cooperation.

Under the joint intentional model, two conspecifics knowingly collaborate in their pursuit of common goal. This requires a shared sense of commitment and necessitates a goal-specific division of labor. That is, for each collaborative activity, each individual has a specific role. According to Tomasello (2014), "this dual-level structure of simultaneous sharedness and individuality—a joint goal but with individual roles—is a uniquely human form of second-person joint engagement requiring species-unique cognitive skills and motivational propensities" (43).

In terms of the representational capacities of early hominids, the joint intentional framework moves beyond the individualistic rendering of situationally relevant content in at least two key ways.

First, the ability to engage in joint intentional activity is predicated on a capacity for joint attention, and such joint attention radically restructures the representational capacity of the “individually intentional” creature. As opposed to merely utilizing imagistic schemata based on its personal history and present individual goal state, the hominid engaging in joint intentional relations has a *perspectival* relation to reality. Crucially, “*the notion of perspective assumes a single target of joint attention on which we have differing perspectives*” (44). And just as joint intentional activity creates a dual-level structure of common goal/individual role, joint attentional activities create a dual-level structure of joint attention and individual perspective. The representational efforts of early hominids thus involve schematizing “a cognitive model of the dual-level collaborative structure comprising a joint goal with individual roles and joint attention with individual perspectives” (69).

Second, perspectival representational capacity expands the conceptual capacity of the previously “individual-intentional” cognizer. The proto-concepts available to great apes, for instance, are always tied to the organism’s individual goal state. A tree, as Tomasello points out, could be representationally construed and categorically registered as an escape route or a sleeping place, but never both at once. Unlike human concepts, which can be overlaid upon the same referent and entertained simultaneously, the ape can only conceptualize an entity based on its particular need or want in a given moment. According to Tomasello, the ability to conceptualize a thing under different aspects comes about as a result of early hominid perspectival cognition. And such was essential for communicative action in collaborative activity:

Indeed, each time [early hominids] communicated, they had to make their communicative act relevant and new for the recipient in the context of her goals and values, their common ground, and her existing knowledge and expectations. As they were thus thinking how their communicative act might fit into the life of the recipient, communicators had to consider several alternative perspectives simultaneously, and only then choose a communicative act to instantiate one of them (loc. cit.).

As Tomasello goes on to underscore, such representational cognition necessarily has to take into account “the conative and epistemic states” of the other (70). In other words, for the first time in evolutionary history, the same situation is cognized with respect to multiple perspectives, thus inaugurating a subject/object divide.

Recall Azeri describes the ideal as having “an objective existence.” The perspectival cognition afforded by joint intentional activity constitutes a giant leap from the subjective and causal cognition of individual intentionality. But have we arrived at the truly objective cognition of “ideal reflection”? To put it another way, while the ape can subjectively entertain and execute protological “if-then” inferences, it certainly falls short of entertaining the “ideal” thought that “ $2+2=4$.” Can such a thought exist for the joint intentional *pre-sapiens* hominin? Not yet, by Tomasello’s account. For that we need *collective intentionality*.

Collective Intentionality and Ideality

Tomasello's joint intentional model provides the necessary pivot away from a primary developmental stage where cognitive capacities are grounded on individual intentionality, to a more complex stage where such representational, communicative, and self-monitoring capacities are restructured and repurposed according to a dyadic interactional framework. It is here that members of the genus *Homo* are first to develop anything close to the abstract conceptual thought and normative cultural roles that come to typify human-specific forms of life. But such a dawning of species-specific cognition consists mostly in ephemeral instances, being tied directly to the joint activity at hand.

With joint intentionality, cognition has become perspectivalized but not yet collectivized, which for Tomasello is the marker of uniquely human culture. Joint intentionality is thus a transitional stage that cognitively and socially scaffolds the emergence of collective intentionality. Where in the joint intentional model an individual coordinates her activity with a specific concrete individual, under the model of collective intentionality an individual coordinates her activity in accordance abstractly, with *any* individual—"some kind of generic other" (Tomasello 2014, 81).

This is not the place to engage in a full recounting of Tomasello's exposition of the development of collective intentionality (which includes detailed discussions of the features of group identification and institutional reality, communicative conventions and resultant conceptualizations, linguistic productivity, and reflective self-monitoring). However, I do want to dwell briefly on Tomasello's account of the origin of reason, which as with Ilyenkov's "ideal," is *normative* in character.

Tomasello notes that while a joint intentional hominin could attempt to convince a collaborative partner of some course of action by, e.g., gesturing to some tracks on the ground that are relevant to the context of the transpiring hunting activity and then pointing to indicate some trajectory that *we* should follow, such communicative activity nonetheless falls short of constituting "fully reasoned thinking" (109). With fully conventional communication, however, "we get to full-blooded reasoning, where 'reasoning' means not just to think about something but to explicate in conventional form—for others or oneself—the reasons why one is thinking what one is thinking" (110).

In this sense, Tomasello follows Hugo Mercier and Dan Sperber in pinpointing the emergence of reasoning within the context of the evolution of human communication and identifying its function as argumentative. According to Mercier and Sperber, signals that reliably demonstrate their own truth abound in the animal kingdom (e.g. antlers and their defensive applicability); however, there are few overt guarantors of the reliability of the "rich and varied informational contents" for creatures engaging in conventional communication (2011, 60). Thus, when there is insufficient trust on the part of the recipient, the communicator uses *reasons* to elicit assent to the veracity of her statement. Tomasello follows all this, but highlights that such exchanges manifest within a cooperative context of shared decision making.

According to Tomasello, “cooperative argumentation in the context of joint or collective decision making is thus premised on a shared metric that we both use in determining which reasons are indeed “best” (Tomasello 2014, 111). Hence, argumentative norms arise and are eventually standardized and even formalized (as early Greek theorists made explicit in the law of noncontradiction and the law of identity). It is also in the argumentative context where once merely informational speech acts become fully assertive speech acts, which commit the speaker not only to honesty but to the *objective* truth of the statement. As such, reasons are provided to make publicly explicit “the bases on which I believe something, which because [others] share these bases, might give them reason to believe it as well” (112). Thus, if it is common knowledge that we avoid certain places with predators, and I have spotted predators at place X today, I have good reason to call into question the plan of foraging at X later in the afternoon. And if someone questions my disapproval of the plan, I only need to make explicit, i.e., assert, that there are predators at X; if challenged I can provide further justifications, but they would not pertain to the common background knowledge that predators are dangerous but rather, e.g., whether I was sure I saw them.

The mechanics of rational argumentation point to the what Davidson (1982) and Searle (1983) describe as the “intrinsically holistic character” of propositional attitudes and intentional states. As Tomasello (2014) explains:

This ability to connect thoughts to other thoughts (both those of others and one’s own) by various inferential relations (prototypically by providing reasons and justifications) is key to human reason in general, and it leads to a kind of interconnection among all of an individual’s potential thoughts in a kind of holistic ‘web of beliefs’ (112).

The Vygotskian uptake of all of this, according to Tomasello (2014), is that it is through internalizing such interpersonal processes of explication and argumentation that an individual can become a rational agent:

Making things explicit to facilitate the comprehension of a recipient leads the communicator to simulate, before actually producing an utterance, how his planned communicative act might be comprehended—perhaps in a kind of inner dialogue. Making things explicit to persuade someone in an argument leads the disputant to simulate ahead of time how a potential opponent might counter his argument, and so to make ready, in thought, an interconnected set of reasons and justifications—again, perhaps, in a kind of inner dialogue.... Human reasoning, even when it is done internally with the self, is therefore shot through and through with a kind of collective normativity in which the individual regulates her actions and thinking based on the group’s normative conventions and standards (112-113).

Tomasello invokes Robert Brandom’s (1994, 497) adage that (to paraphrase) “monological thinking is parasitic on dialogical thinking,” but the Vygotskian point remains—particularly human cognition is founded on interpersonal social processes.

What does this mean for mathematical cognition? Hopefully we've seen that mere hypothetical inferences do not provide the adequate foundation for the development of normative ideal cognition. Collective communicative conventions must be in place first in order that any *metacognitive* thought "X is correct" or "X is incorrect" can ever come about. A thought such "2+2=5 is incorrect" is thus only possible on the basis of collective intentionality.

Final Remarks

This has hopefully been an informative backstory to the emergence of the abstract normative cognition requisite for mathematical reasoning. Unfortunately, all this has not done much to answer Azeri's concluding questions regarding the rise of mathematics' "quantitative attitude toward reality" and how this might have accompanied the Modern shift toward Capitalist relations of production. One could hope that Vygotsky, Leontiev, and Tomasello might provide some of tools needed to answer this. Indeed, Azeri already has much of the theoretical footing in place for what must follow as an interdisciplinary investigation. I only hope to have contributed to that foundation.

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